PHYSICAL REVIEW E **77**, 056609 (2008)

Negative index of refraction in metallic metamaterial comprising split-ring resonators

Zheng-Gao Dong,[1,*] Shuang-Ying Lei,[2] Ming-Xiang Xu,[1] Hui Liu,[3] Tao Li,[3] Fu-Ming Wang,[3] and Shi-Ning Zhu[3,†]
[1]*Physics Department, Southeast University, Nanjing 211189, People's Republic of China*
[2]*Key Laboratory of MEMS of Ministry of Education, Southeast University, Nanjing 210096, People's Republic of China*
[3]*National Laboratory of Solid State Microstructures, Nanjing University, Nanjing 210093, People's Republic of China*

(Received 2 January 2008; revised manuscript received 16 March 2008; published 29 May 2008)

We numerically investigate the negative index of refraction in a metamaterial composed of metallic split-ring resonators, which exhibits simultaneously negative permittivity and permeability without resorting to additional metallic wires. It is confirmed that, in the left-handed band, negative permittivity is generated in analogy to the cut-wire metamaterial and negative permeability comes from the antisymmetric resonant mode, which occurs at a frequency band about 3 times higher than the fundamental magnetic resonance proposed by Pendry*et al.* [IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999)].

DOI: 10.1103/PhysRevE.77.056609 PACS number(s): 41.20.Jb, 42.70.Qs, 78.20.Ci

I. INTRODUCTION

Metallic metamaterials comprising double split-ring resonators (SRRs) are the main artificial structures to realize magnetic responses at an electromagnetic spectrum above gigahertz frequencies [1–4]. It is of great interest and importance because significant magnetism from SRRs can be generated at high frequencies where no naturally occurring material is found to reveal a magnetic response. As for those subsequently proposed magnetic metamaterials, such as the single split-ring resonators [5], U-shaped patterns [6], cut-wire pairs [7,8], fishnet structures [9,10], and split-ring chains [11], they generally share the common mechanism of magnetic response. Generally speaking, the circulating currents induced in the conducting elements with splits result in resonant permeability (μ_{eff}) in certain frequencies, which usually can reach negative values and, thus, are prepared for designs of left-handed materials (LHMs).

Casse *et al.* published a magnetic resonance frequency of 187.5 THz for the SRR design [12,13], but the applicability of the magnetic response from SRR is experimentally hard to realize in significantly higher frequencies because the state-of-the-art technology cannot shrink down the scale of SRRs as small as desired. Therefore, as far as the SRR with a given scale is concerned, it is of obvious advantage, experimentally as well as theoretically, to explore the possible magnetic response at higher frequency than its fundamental magnetic resonant frequency, proposed by Pendry *et al.* [1]. On the other hand, it is generally believed that, in addition to the capability of a negative magnetic response ($\mu_{\mathrm{eff}}<0$), the SRR should be of capability to produce a negative electric response ($\varepsilon_{\mathrm{eff}}<0$) at certain frequencies in analogy to metamaterial comprising finite-length wires (i.e., cut wires). Consequently, a metamaterial composed of SRRs could be an LHM when the negative electric and magnetic responses are modulated to a common frequency band. However, this is typically difficult to design [14,15]. As a matter of fact, there is no LHM composed of SRR units claimed yet.

In this work, we numerically investigate metallic SRRs to confirm their left-handed response with simultaneously negative μ_{eff} and $\varepsilon_{\mathrm{eff}}$. It is stressed that the proposed LHM comprising SRRs is based on the magnetic resonant mode at higher frequency (about a factor of 3) than its fundamental magnetic resonant frequency, proposed by Pendry *et al.* For convenience, the magnetic resonant mode of the former is hereafter called the antisymmetric resonant mode and the latter is called the fundamental resonant mode since it is extensively referred to in the literature [1–6].

II. SIMULATION RESULTS

A. Numerical model and transmission spectrum

The unit cell of the metallic SRR metamaterial is schematically shown in Fig. 1. The split rings are copper in square with a thickness of 0.02 mm, and the geometric parameters a and b are 3.0 and 0.2 mm, respectively. The gap between the inner and outer rings, represented by g, is 0.1 mm, and each of the splits in the inner and outer rings has

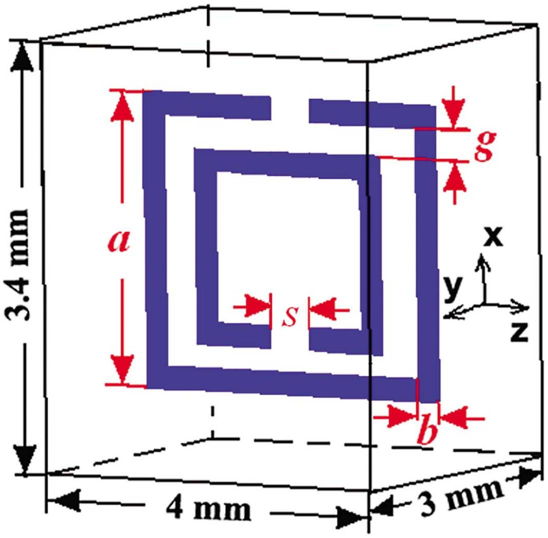


FIG. 1. (Color online) The unit cell of the metallic metamaterial comprising SRRs.

---

[*] zgdong@seu.edu.cn
[†] zhusn@nju.edu.cn

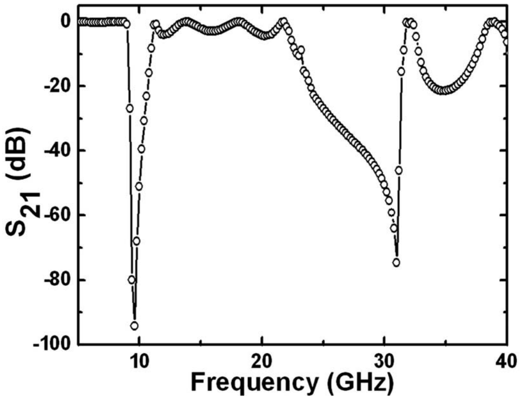


FIG. 2. The transmission coefficient of the metallic metamaterial comprising SRRs.

the same width *s* of 0.6 mm. In our numerical simulations based on the full-wave finite-element method, the simulation configuration has a dimension of $1\times1\times8$ units (i.e., one unit in the transverse *xy* plane and eight units in the electromagnetic propagation direction along the *z* axis). The polarized incident wave with electric field in the *x* direction and magnetic field in the *y* direction is satisfied by applying perfect electric and magnetic boundary conditions, respectively [16,17]. For simplicity, all simulations are performed in the background of vacuum without taking the substrate into account.

The transmission coefficient (scattering parameter S_{21}) of the double-ring metamaterial is shown in Fig. 2. It is obvious that there are a stop band near 9.4 GHz and a passband around 32 GHz, both of which are the bands of our main objective of investigation. As will be confirmed in this work, the stop band corresponds to the fundamental resonant mode of magnetic response, proposed by Pendry *et al.*, while the passband is a left-handed transmission band associated with a magnetic resonance with antisymmetric currents induced in the neighboring edges of the rings.

B. Fundamental resonant mode near 9.4 GHz

To confirm that the stop band near 9.4 GHz is attributed to negative $\boldsymbol{\mu}_{\text{eff}}$ with the magnetically resonant mode proposed by Pendry *et al.*, we can additionally set metallic continuous wires beside the SRRs to find what happens to the transmission spectrum. In our simulations, the continuous wires are copper with width of 0.6 mm. From the result shown in Fig. 3, there is a passband near 9.4 GHz, which otherwise should be a stop band (see Fig. 2) if no continuous wires are considered. These transmission results are consistent with works in the literature [18,19] involving the magnetic response of the SRRs, proposed by Pendry *et al.* In the next section, we will confirm another magnetic resonance in the form of left-handed passband with the same SRR configuration at frequencies around 32 GHz (see Fig. 2), which is approximately 3 times higher than the left-handed band with fundamental resonant mode.

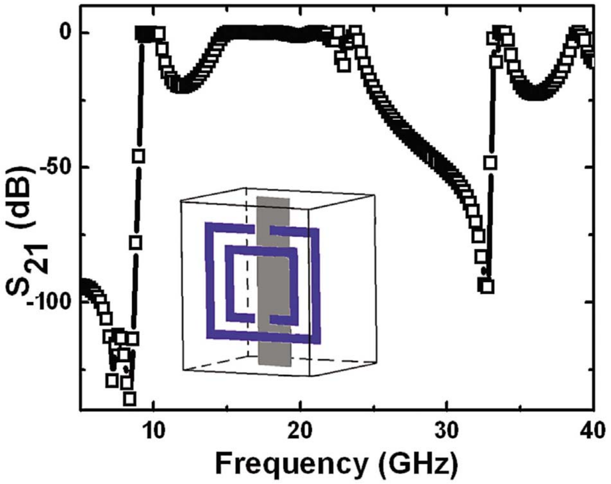


FIG. 3. (Color online) The transmission coefficient of the metamaterial combined by SRRs and continuous wires.

C. Antisymmetric resonant mode around 32 GHz

Before the investigation of the magnetic response with antisymmetric resonant mode from the SRRs, we simulate a square-plate metamaterial with cross section $a\times a$, which should eliminate the possible magnetic response of the SRRs while keeping its electric response to a sufficient extent. As shown in Fig. 4, the transmission spectrum of the square-plate metamaterial, which essentially resembled a cut-wire metamaterial, shows that there is a stop band with negative $\boldsymbol{\varepsilon}_{\text{eff}}$ from about 22 to 42 GHz, corresponding to the electric resonance frequency ($\boldsymbol{\omega}_{\text{eo}}$) and electric plasma frequency ($\boldsymbol{\omega}_{\text{ep}}$), respectively. By comparing this stop band with the result in Fig. 2, it is indicated that the passband of the SRRs around 32 GHz (Fig. 2) occurs within the negative $\boldsymbol{\varepsilon}_{\text{eff}}$ frequency band, approximately from 22 to 39 GHz.

On the other hand, it is also confirmed in this work that the passband of the SRRs around 32 GHz is associated with a magnetically resonant response with negative $\boldsymbol{\mu}_{\text{eff}}$. Generally speaking, the magnetic metamaterials with structured metallic elements, such as the SRRs and cut-wire pairs, could reveal negative magnetic responses when circulating currents, including the antiparallel currents (antisymmetric resonant mode; see Refs. [14,20]), are induced at certain fre-

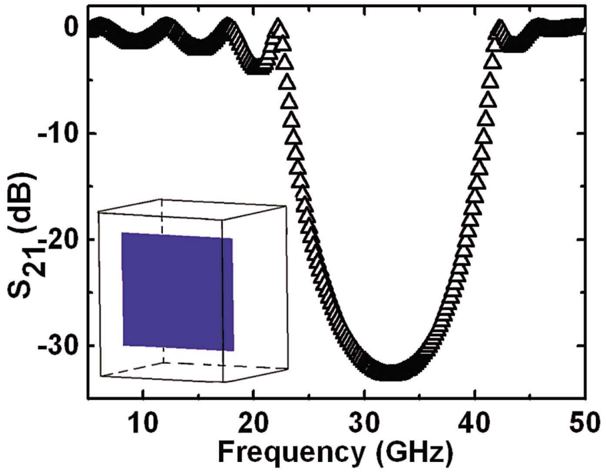
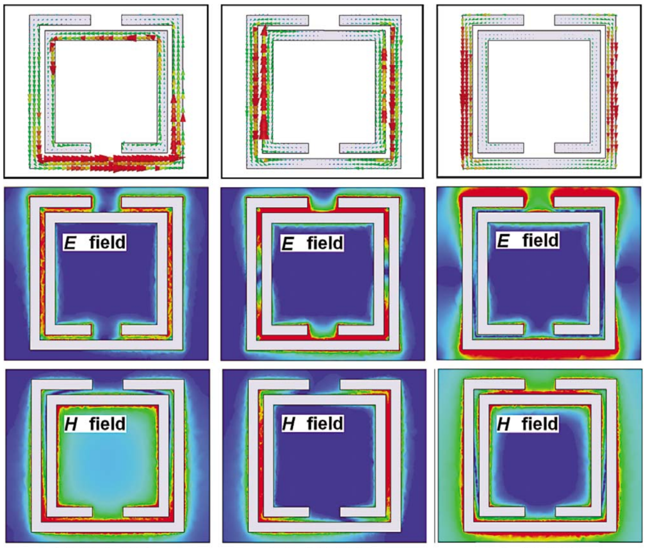


FIG. 4. (Color online) The transmission coefficient of the square-plate metamaterial.



(a) 9.4 GHz (b) 32 GHz (c) 18 GHz

FIG. 5. (Color online) The distributions of the induced currents and magnitude maps of the electromagnetic field. (a) The left panel at 9.4 GHz is the fundamental resonant mode. (b) The middle panel at 32 GHz is the antisymmetric resonant mode. (c) The right panel at 18 GHz is the nonresonant case.

quencies. Figure 5 shows the induced current distributions and the electromagnetic field maps at three frequencies: namely, 9.4 GHz in the fundamental resonant band, 32 GHz in the antisymmetric resonant band, and 18 GHz in the nonresonant band. It is obvious that the circulating currents are induced in the fundamental resonant mode (the left panel of Fig. 5, at 9.4 GHz). As for the antisymmetric resonant mode, antiparallel currents are induced between the neighboring edges of the inner and outer split rings (the middle panel of Fig. 5, at 32 GHz), which are contributing to negative μ_{eff} [14,20]. In contrast, at the nonresonant transmission band, neither circulating nor antiparallel currents are formed and kept in phase (the right panel of Fig. 5, at 18 GHz). These results imply that both the stop band near 9.4 GHz and the passband around 32 GHz are involved in magnetic resonances. As for the former, it exhibits as a stop band because the fundamental magnetic resonance of the SRRs occurs at the positive $\varepsilon_{\mathrm{eff}}$ regime. As for the latter, it exhibits as a passband because the magnetic resonance with antisymmetric mode is happened within the negative $\varepsilon_{\mathrm{eff}}$ range of the SRRs. Accordingly, simultaneously negative $\varepsilon_{\mathrm{eff}}$ and μ_{eff} can be expected in this transmission band, where the resonant frequency is about 3 times higher than that of its corresponding SRR-wire LHM with the same SRR scale. In addition, it is worth mentioning that, for the antisymmetric resonance, the magnetic field is localized in the left and right gaps between the inner and outer rings, while the electric field is localized in the upper and lower gaps, whereas for the fundamental resonant mode the localized field is nearly distributed homogeneously along the entire gaps and for the nonresonant case no strong field is localized in the gaps, expect a slight magnitude enhancement between the adjacent SRR structures.

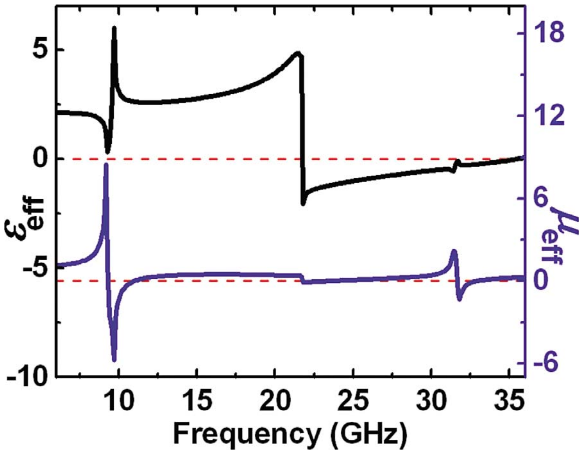


FIG. 6. (Color online) The retrieved permittivity and permeability of the metamaterial comprising SRRs.

D. Retrieved constitutive parameters

For further clarity, a retrieval calculation was performed to obtain the $\varepsilon_{\mathrm{eff}}$ and μ_{eff} from the scattering parameters (S_{21} and S_{11}) of the proposed SRR metamaterial [21,22]. The retrieved results in Fig. 6 reveal that simultaneously negative $\varepsilon_{\mathrm{eff}}$ and μ_{eff}, corresponding to the antisymmetric resonant mode, are obtained around 32 GHz from the SRRs. It is also obvious that the retrieved negative μ_{eff} and positive $\varepsilon_{\mathrm{eff}}$ in the fundamental resonant regime are contributing to the stop band near 9.4 GHz. Therefore, all the retrieved data are consistent with the results concluded earlier.

III. CONCLUSION

We numerically investigate the left-handed response in metallic metamaterial comprising SRRs. It is found that, in addition to the fundamental magnetic resonance proposed by Pendry *et al.*, the SRRs can generate an antisymmetric resonant mode with negative μ_{eff}, attributed to the antiparallel currents induced in the neighboring edges of the split rings. Based on this antisymmetric resonant mode, an LHM comprising SRRs, without using additional wires, can be realized under suitable geometric parameters of the SRR configuration.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Contract Nos. 10747116, 10534020, 60578034, 10704036, and 10604029 and the State Key Program for Basic Research of China (Grant No. 2006CB921804). M.-X.X. also acknowledges support from the National Science Foundation of Jiangsu Province of China (Grant No. BK2007118) and the Cyanine-Project Foundation from Jiangsu Province of China (Grant No. 1107020060).

[1] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).

[2] T. J. Yen, W. J. Padilla, N. Fang, D. C. Vier, D. R. Smith, J. B. Pendry, D. N. Basov, and X. Zhang, Science **303**, 1494 (2004).

[3] H. O. Moser, B. D. F. Casse, O. Wilhelmi, and B. T. Saw, Phys. Rev. Lett. **94**, 063901 (2005).

[4] B. D. F. Casse, H. O. Moser, J. W. Lee, M. Bahou, S. Inglis, and L. K. Jian, Appl. Phys. Lett. **90**, 254106 (2007).

[5] S. Linden, C. Enkrich, M. Wegener, J. Zhou, T. Koschny, and C. M. Soukoulis, Science **306**, 1351 (2004).

[6] C. Enkrich, M. Wegener, S. Linden, S. Burger, L. Zschiedrich, F. Schmidt, J. F. Zhou, Th. Koschny, and C. M. Soukoulis, Phys. Rev. Lett. **95**, 203901 (2005).

[7] V. M. Shalaev, W. Cai, U. K. Chettiar, H.-K. Yuan, A. K. Sarychev, V. P. Drachev, and A. V. Kildishev, Opt. Lett. **30**, 3356 (2005).

[8] J. Zhou, L. Zhang, G. Tuttle, T. Koschny, and C. M. Soukoulis, Phys. Rev. B **73**, 041101(R) (2006).

[9] S. Zhang, W. Fan, K. J. Malloy, S. R. J. Brueck, N. C. Panoiu, and R. M. Osgood, Opt. Express **13**, 4922 (2005).

[10] M. Kafesaki, I. Tsiapa, N. Katsarakis, Th. Koschny, C. M. Soukoulis, and E. N. Economou, Phys. Rev. B **75**, 235114 (2007).

[11] H. Liu, D. A. Genov, D. M. Wu, Y. M. Liu, J. M. Steele, C. Sun, S. N. Zhu, and X. Zhang, Phys. Rev. Lett. **97**, 243902 (2006).

[12] B. D. F. Casse, H. O. Moser, L. K. Jian, M. Bahou, O. Wilhelmi, B. T. Saw, and P. D. Gu, J. Phys.: Conf. Ser. **34**, 885 (2006).

[13] A higher magnetic resonance for the SRR design was experimentally reached at 216 THz; see B. D. F. Casse, Ph.D. thesis, National University of Singapore, 2007.

[14] V. M. Shalaev, Nat. Photonics **1**, 41 (2007).

[15] C. M. Soukoulis, S. Linden, and M. Wegener, Science **315**, 47 (2007).

[16] F. M. Wang, H. Liu, T. Li, Z. G. Dong, S. N. Zhu, and X. Zhang, Phys. Rev. E **75**, 016604 (2007).

[17] Z. G. Dong, S. N. Zhu, H. Liu, J. Zhu, and W. Cao, Phys. Rev. E **72**, 016607 (2005).

[18] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).

[19] M. Bayindir, K. Aydin, E. Ozbay, P. Markos, and C. M. Soukoulis, Appl. Phys. Lett. **81**, 120 (2002).

[20] A. N. Grigorenko, A. K. Geim, H. F. Gleeson, Y. Zhang, A. A. Firsov, I. Y. Khrushchev, and J. Petrovic, Nature (London) **438**, 335 (2005).

[21] D. R. Smith, D. C. Vier, Th. Koschny, and C. M. Soukoulis, Phys. Rev. E **71**, 036617 (2005).

[22] X. Chen, T. M. Grzegorczyk, B.-I. Wu, J. Pacheco, and J. A. Kong, Phys. Rev. E **70**, 016608 (2004).